Prof. Hadley

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THE

MATHEMATICAL MONTHLY.

JANUARY, 1861.

EDITED BY

J. D. RUNKLE, A.M., A.A.S.

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Editorial Items.

Prizes for Vol. III. offered to all Students.

For Solutions. — A 1st Prize, $7.50; 2d, $5; 3d, $3; 4th, Vol. III. The third and fourth prizes are confined to beginners for solutions of Problems I. and II.; but as the best solutions only will be published, advanced students are desired to send solutions of all the problems.

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A Generous Offer.

It gives us great pleasure to announce that "A Friend of the Mathematical Monthly Prizes," whose name we are not permitted to give, wishes to assume the responsibility of the Prizes for the solutions of the Problems in the number for December. He says, "The Monthly has been of great value to me. It has been my tutor, and certainly the tuition is cheap. I ought to be able now to contribute something more to its sustenance." The solutions will be due February 1, 1861, and the Report will be published in the March number. We hope to receive a large number of solutions as evidence of an exciting contest for these Prizes. It will be seen (Editorial Items), that the times for the solutions of the September and October Problems are extended one month. Hereafter we shall issue the Monthly promptly on or before the first of the month.

Portrait of Sir John Herschel.

Through the kindness of Miss Maria Mitchell we have received a fine photograph of Sir John Herschel, which we shall have engraved on steel for an early number.

We expect our friends will aid us by increasing our subscription list. See Mathematical Monthly, on seventh page of Advertiser.

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THE TRIBUNE FOR 1861.

PROSPECTUS.

The XXth Volume of THE WEEKLY TBIBUNE commenced with the issue of September 1. During the past year THE TRIBUNE has been obliged to devote quite a large proportion of its space to Politics, but we shall soon be able to forego political discussion almost entirely, for months if not for years, and devote nearly all our columns to subjects of less intense, but more abiding, interest. Among these, we mean to pay especial attention to

I. EDUCATION. — The whole subject of Education, both Popular and General, will be discussed in our columns throughout the year 1861, and we hope to enlist in that discussion some of the profoundest thinkers and the ablest instructors in our country. It is at once our hope and our resolve that the cause of Education shall receive an impetus from the exertions of THE TRIBUNE in its behalf during the year 1861.

II. AGRICULTURE. — We have been compelled to restrict our elucidations of this great interest throughout 1860, and shall endeavor to atone therefor in 1861. Whatever discovery, deduction, demonstration, is calculated to render the reward of labor devoted to cultivation more ample or more certain, shall receive prompt and full attention.

III. MANUFACTURES, &c. — We hail every invention or enterprise whereby American Capital and Labor are attracted to and advantageously employed in any department of Manufacturing or Mechanical Industry as a real contribution to the Public Weal, insuring ampler, steadier, more convenient, more remunerating markets to the Farmer, with fuller employment and better wages to the Laborer. The Progress of Mining, Iron-making, Steel-making, Cloth-weaving, &c., &c., in our country, and the world, shall be watched and reported by us with an earnest and active sympathy.

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THE

MATHEMATICAL MONTHLY.

Vol. III. . . . JANUARY, 1861. . . . No. IV.

PRIZE PROBLEMS FOR STUDENTS.

I. $ABCD$ is a square, AC its diagonal, bisect AD in E, join BE cutting AC in F; then will

$$\frac{\triangle AEF}{1} = \frac{\triangle CEF}{2} = \frac{\triangle ABE}{3} = \frac{\triangle BCF}{4}.$$

II. If $\frac{a-b}{ay+bx} = \frac{b-c}{bz+cx} = \frac{c-a}{cy+az} = \frac{a+b+c}{ax+by+cz}$; then each of these fractions $= \frac{1}{x+y+z}$.

III. If d be the perpendicular, dropped from the angle C on the opposite side of a triangle, show that $\sin C = \frac{c\,d}{a\,b}$. Deduce the formula

$$\sin(A+B) = \sin A \cos B + \cos A \sin B.$$

IV. An hyperbola, having its asymptotes parallel to the axes of co-ordinates, may be made to pass through the angular points of the quadrilateral formed by the four lines whose equations are

(1) $y + x + 6a = 0$,
(2) $y + 2x + 3a = 0$,
(3) $y + 6x + a = 0$,
(4) $y + 3x + 2a = 0$.

V. Three lines, x, y, z, are drawn from the angular points of a triangle to a point within it. Show that when $x^n + y^n + z^n$ is a minimum we have

$$\frac{\sin(x,y)}{x^{n-1}} = \frac{\sin(x,z)}{y^{n-1}} = \frac{\sin(y,z)}{z^{n-1}},$$

in which (x, y) denotes the angle between x and y.

Solutions of these problems must be received by March 1, 1861.

NOTES AND QUERIES.

1. *To express the sines and cosines of the sum and difference of two angles in terms of the sines and cosines of the angles themselves.*

Let O be the origin of rectangular co-ordinates, and the pole of polar co-ordinates, the axis of x being the polar axis; then the square of the distance between two points expressed in polar co-ordinates (r, φ), (r', φ'), and in rectangular co-ordinates (x, y), (x', y'), is

$$(1) \qquad r^2 + r'^2 - 2rr' \cos(\varphi - \varphi') = (x - x')^2 + (y - y')^2.$$

But these two systems of co-ordinates are connected by the equations

$$r^2 = x^2 + y^2, \qquad x = r\cos\varphi, \qquad y = r\sin\varphi,$$
$$r'^2 = x'^2 + y'^2, \qquad x' = r'\cos\varphi', \qquad y' = r'\sin\varphi',$$

and therefore (1) reduces to

$$(2) \quad \cos(\varphi - \varphi') = \frac{x}{r}\cdot\frac{x'}{r'} + \frac{y}{r}\cdot\frac{y'}{r'} = \cos\varphi\cos\varphi' + \sin\varphi\sin\varphi'.$$

If we change the sign of φ', then $\sin\varphi'$, or y', will change its sign; therefore

$$(3) \quad \cos(\varphi + \varphi') = \frac{x}{r}\cdot\frac{x'}{r'} - \frac{y}{r}\cdot\frac{y'}{r'} = \cos\varphi\cos\varphi' - \sin\varphi\sin\varphi'.$$

From (1) we get

$$rr'\cos(\varphi - \varphi') = xx' + yy';$$
$$\therefore r^2r'^2 - r^2r'^2\cos^2(\varphi - \varphi') = r^2r'^2 - (xx' + yy')^2,$$
$$\therefore r^2r'^2\sin^2(\varphi - \varphi') = x^2y'^2 - 2xx'yy' + x'^2y^2,$$
$$\therefore rr'\sin(\varphi - \varphi') = xy' - yx',$$
$$\therefore \sin(\varphi - \varphi') = \frac{x}{r}\cdot\frac{y'}{r'} - \frac{y}{r}\cdot\frac{x'}{r'} = \sin\varphi\cos\varphi' - \cos\varphi\sin\varphi',$$

and $$\sin(\varphi + \varphi') = \frac{x}{r}\cdot\frac{y'}{r'} + \frac{y}{r}\cdot\frac{x'}{r'} = \sin\varphi\cos\varphi' + \cos\varphi\sin\varphi'.$$

2. *In a spherical triangle, to express the cosine of an angle in terms of sines and cosines of the sides.*

Let ABC be any spherical triangle, O the centre of the sphere, being the origin of co-ordinates, and OC the axis of z; let r denote the radius of the sphere, and (x, y, z), (x', y', z') the co-ordinates of A and B respectively; then the square of the chord AB gives

$$(1) \quad r^2 + r^2 - 2r^2 \cos AOB = (x-x')^2 + (y-y')^2 + (z-z')^2.$$

But these two systems of co-ordinates are connected by the equations

$$\begin{aligned} r^2 &= x^2 + y^2 + z^2, & r^2 &= x'^2 + y'^2 + z'^2, \\ x &= r \sin\theta \cos\varphi, & x' &= r \sin\theta' \cos\varphi', \\ y &= r \sin\theta \sin\varphi, & y' &= r \sin\theta' \sin\varphi', \\ z &= r \cos\theta, & z' &= r \cos\theta'; \end{aligned}$$

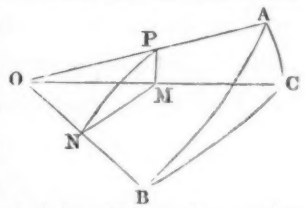
and (1) reduces to

$$\begin{aligned} \cos AOB &= \frac{x}{r}\cdot\frac{x'}{r} + \frac{y}{r}\cdot\frac{y'}{r} + \frac{z}{r}\cdot\frac{z'}{r}. \\ &= \sin\theta \sin\theta' \cos\varphi \cos\varphi' + \sin\theta \sin\theta' \sin\varphi \sin\varphi' + \cos\theta \cos\theta' \\ &= \sin\theta \sin\theta' (\cos\varphi \cos\varphi' + \sin\varphi \sin\varphi') + \cos\theta \cos\theta' \\ &= \sin\theta \sin\theta' \cos(\varphi - \varphi') + \cos\theta \cos\theta'; \end{aligned}$$

or $\cos c = \sin a \sin b \cos C + \cos a \cos b.$

The method of proof used in these notes is perfectly general, since all the equations used are universally true.

3. *Note on* NAPIER'S *Rules of Circular Parts.* — The formulas for the solution of right-angled spherical triangles are very easily deduced, as follows: Let the triangle ABC be right angled at C, and O be the centre of the sphere. Take any point, P, in AO; draw PM perpendicular to OC, MN perpendicular to OB, and join NP. Since the planes AOC and BOC are perpendicular to each other,



$$\therefore PN^2 = PM^2 + MN^2 = OP^2 - OM^2 + OM^2 - ON^2 = OP^2 - ON^2;$$

therefore PNO is a right angle. Hence

(1) $\frac{ON}{OP} = \frac{ON}{OM} \cdot \frac{OM}{OP}$; i. e. $\cos c = \cos a \cos b$.

(2) $\frac{PM}{OP} = \frac{PM}{PN} \cdot \frac{PN}{OP}$; i. e. $\sin b = \sin B \sin c$, $\sin a = \sin A \sin c$.

(3) $\frac{MN}{ON} = \frac{MN}{PN} \cdot \frac{PN}{ON}$; i. e. $\tan a = \cos B \tan c$, $\tan b = \cos A \tan c$.

(4) $\frac{PM}{OM} = \frac{PM}{MN} \cdot \frac{MN}{OM}$; i. e. $\tan b = \tan B \sin a$, $\tan a = \tan A \sin b$.

The last equations in (2), (3), (4), follow by a simple interchange of letters.

From the product of the second and third equations of (4) we get

$$\tan A \tan B = \frac{\tan a \tan b}{\sin a \sin b} = \frac{1}{\cos a \cos b} = \frac{1}{\cos c} \text{ by (1).}$$

(5) $$\therefore \cos c = \cot A \cot B.$$

Multiply crosswise the third equation of (2) and the second of (3); we obtain

$$\sin a \cos B \tan c = \tan a \sin A \sin c.$$

$$\therefore \cos B = \frac{\sin A \cos c}{\cos a} = \sin A \cos b \quad \text{by (1).}$$

(6) $$\therefore \cos B = \sin A \cos b, \qquad \cos A = \sin B \cos a.$$

Out of the five quantities a, b, c, A, B, only ten combinations of three can be formed; and, therefore, the ten equations just found solve all the cases of right-angled spherical triangles. Any two of the five quantities being given, and a third required, some one of these ten equations will give the required quantity in terms of the given ones.

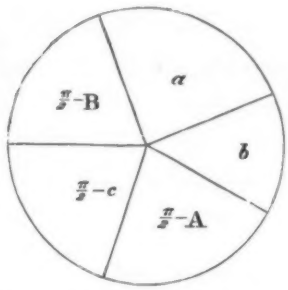
These ten equations are comprised in the six following corresponding theorems, since four of them apply twice to the same triangle.

(1) cos hyp = product of cosines of sides.

(5) cos hyp = products of cotangents of angles.

(2) sin side = sine of opposite angle × sin hyp.
(3) tan side = tan hyp × cos included angle.
(4) tan side = tan opposite angle × sine of other side.
(6) cos angle = cos opposite side × sine of other angle.

These six formulas may be condensed into two rules, known as NAPIER'S *Rules of Circular Parts*, which were first published by him in a work entitled *Mirifici Logarithmorum Canonis Descriptio.* Edinburgh, 1614.



Omitting the right angle, the circular parts are the two sides, including the right angle, and the complements of the hypothenuse and the two other angles; thus, a, b, $\frac{\pi}{2}-A$, $\frac{\pi}{2}-c$, $\frac{\pi}{2}-B$, are the five circular parts, and selecting any one of the five as the *middle part*, two of the remaining four are *adjacent*, and the other two *opposite* to the middle part. Then NAPIER'S two rules are

sine of middle part = product of tangents of adjacent parts.
sine of middle part = product of cosines of opposite parts.

These rules are memorized for use by associating *sine* and *middle*, *tangent* and *adjacent*, *cosine* and *opposite*.

The following table shows that these two rules involve the ten cases already found; the middle part stands first, the adjacent parts next, and the opposite parts in the third member of the equations.

$\sin a = \tan b \tan\left(\frac{\pi}{2}-B\right) = \cos\left(\frac{\pi}{2}-A\right)\cos\left(\frac{\pi}{2}-c\right)$, by Rules;
$\sin a = \tan b \cot B = \sin A \sin c$. Reduced.
$\sin b = \tan a \tan\left(\frac{\pi}{2}-A\right) = \cos\left(\frac{\pi}{2}-B\right)\cos\left(\frac{\pi}{2}-c\right)$, by Rules;
$\sin b = \tan a \cot A = \sin B \sin c$. Reduced.

$$\sin\left(\frac{\pi}{2}-A\right)=\tan b\tan\left(\frac{\pi}{2}-c\right)=\cos a\cos\left(\frac{\pi}{2}-B\right),\qquad\text{by Rules;}$$

$$\cos A=\tan b\cot c\qquad=\cos a\sin B.\qquad\text{Reduced.}$$

$$\sin\left(\frac{\pi}{2}-c\right)=\tan\left(\frac{\pi}{2}-A\right)\tan\left(\frac{\pi}{2}-B\right)=\cos a\cos b,\qquad\text{by Rules;}$$

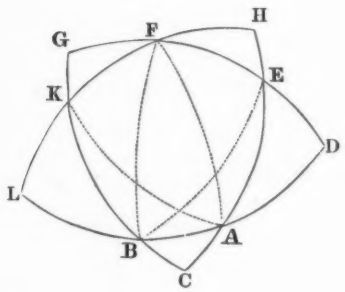
$$\cos c=\cot A\cot B\qquad=\cos a\cos b.\qquad\text{Reduced.}$$

$$\sin\left(\frac{\pi}{2}-B\right)=\tan a\tan\left(\frac{\pi}{2}-c\right)=\cos b\cos\left(\frac{\pi}{2}-A\right),\qquad\text{by Rules;}$$

$$\cos B=\tan a\cot c\qquad=\cos b\sin A.\qquad\text{Reduced.}$$

Besides this method of proving these rules, NAPIER has indicated another, on pp. 32–35 of the work already referred to, which is as follows: —

Let ABC be a spherical triangle right-angled at C. With A and B as poles describe great circles, and produce all the sides of ABC to meet them. The five triangles ABC, ADE, EHF, FGK, KLB are all right angled, and we propose to show that they all have the *same circular parts*. Let us first consider ABC and ADE. Denote the elements of ABC, as usual, by a, b, c, A, B; then, in the triangle ADE, the angle $EAD=A$; the side AD is the complement of AB, and equals $\frac{\pi}{2}-c$; the side DE equals $\frac{\pi}{2}-B$, because it measures the angle EBD, which is the complement of $ABC=B$; the angle DEA equals $\frac{\pi}{2}-a$, because it is the complement of the angle BEC, which is measured by $BC=a$; the side EA is the complement of AC, and equals $\frac{\pi}{2}-b$. The elements of ADE are therefore $\frac{\pi}{2}-c$, $\frac{\pi}{2}-B$, $\frac{\pi}{2}-b$, $\frac{\pi}{2}-a$, A, and its circular parts are $\frac{\pi}{2}-c$, $\frac{\pi}{2}-B$, b, a, $\frac{\pi}{2}-A$, which are those of ABC.



It can be shown in the same way that the triangles AED and EHF have the same circular parts, and so on.

The following table exhibits in one view all the elements of these five allied triangles, beginning with the hypothenuse, and numbering around to the right.

	ABC	ADE	EHF	FGK	KLB
1.	$AB=c$	$AE=\frac{\pi}{2}-b$	$FE=B$	$KF=A$	$BK=\frac{\pi}{2}-a$
2.	$BAC=A$	$DEA=\frac{\pi}{2}-a$	$HEF=c$	$GKF=\frac{\pi}{2}-b$	$LBK=B$
3.	$CA=b$	$DE=\frac{\pi}{2}-B$	$HF=\frac{\pi}{2}-A$	$GK=a$	$LB=\frac{\pi}{2}-c$
4.	$BC=a$	$AD=\frac{\pi}{2}-c$	$EH=b$	$FG=\frac{\pi}{2}-B$	$KL=\frac{\pi}{2}-A$
5.	$ABC=B$	$EAD=A$	$HEF=\frac{\pi}{2}-a$	$KFG=c$	$LKB=\frac{\pi}{2}-b$

The complements of the 3d, 4th, 5th, and 1st elements of any one of these triangles are the 1st, 2d, 3d, and 4th elements of the right-hand adjacent triangle, while their 2d and 5th elements are the same; and by applying this law to the five triangles in succession, we shall come back to the set of elements from which we started.

If we apply theorems (1) and (5) to each of these five triangles, or either of the remaining four theorems twice to each one of them, or all the theorems to any one of them, we shall obtain the ten original equations.

It follows, therefore, since the circular parts of these triangles are the same, that we obtain the same results by applying the two *Rules of Circular Parts* to each of the five circular parts of the same triangle, as by applying the theorems to the allied triangles, either all to one, or two to each of the five in succession.

This method considers each rule, not as the statement of dissimilar properties of one triangle, but as the statement of similar properties of five allied triangles. These notes on Spherical Trigonometry are substantially from TODHUNTER'S excellent work on this subject.

REPORT OF THE JUDGES UPON THE SOLUTIONS OF THE PRIZE PROBLEMS IN No. XII., Vol. II.

The first Prize is awarded to Asher B. Evans, Madison Univ., N. Y.

The second prize is not awarded.

The third prize is awarded to John M. Terrill, Linneus, Mo.

Prize Solution of Problem I.

By Miss H. S. Hazeltine, Worcester, Mass.

If AB, CD be chords of a circle at right angles to each other, prove that the sum of the arcs AC, BD is equal to half the circumference.

Designate by E the point where the chords intersect each other, and draw the chord AC. In the right triangle AEC, the sum of the acute angles is 90°; whence

$$\text{arc } BC + \text{arc } DD = \text{semi-circumference} = \text{arc } AC + \text{arc } BD,$$

or $180° - (\text{arc } AC + \text{arc } BD)$, according as E falls outside or inside of the circle.

Prize Solution of Problem II.

By John M. Terrill, Linneus, Mo.

Prove that the cube of any number and the number itself, being divided by 6, leave the same remainder.

Take $a = 6b + c$. Then $a^3 = 216\,b^3 + 108\,b^2 c + 18\,b\,c^2 + c^3$. In dividing this last expression by 6, if there be a remainder it must be found in the last term, c^3, because all the other terms are multiples of 6. This remainder is found by trial to be equal to c for all values of the latter less than 6, that is, for all possible values of c; that is, when c is 1, 2, 3, 4, or 5, the remainders of c^3 divided by 6 are 1, 2, 3, 4, 5.

Second Solution.

By Miss Mary A. Willis, Nunda, N. Y.

We know (Math. Monthly, Vol. II., No. XI.) that $x^3 - x$ is divisible by 6 when x is a whole number. Therefore, since $x^3 = 6q + r$

and $x = 6q' + r'$, in which q, q' are the quotients, and r, r' the remainders, the equation

$$x^3 - x = 6(q - q') + r - r'$$

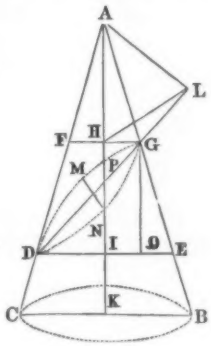
must be divisible by 6; and therefore $r - r'$ is divisible by 6, which cannot be the case, since $r - r' < 6$. Therefore $r - r' = 0$, or $r = r'$, as was to be proved.

Prize Solution of Problem III.

By Asher B. Evans, Madison University, Hamilton, N. Y.

It is required to divide a given right cone into two parts, having the ratio of m to n, by a plane making an angle of p degrees with the axis.

Draw FG and DE parallel to the base BC and cutting the axis of the cone in H and I; draw AL perpendicular to DG produced, and join H and L; then, if MN be the diameter, conjugate to DG in the elliptical section $GMDN$, MN will be equal to $\sqrt{(DE \cdot FG)}$. The part ADG, above the elliptical section is an oblique elliptical cone, and



$$\frac{AL}{3} \cdot DG \cdot MN \frac{\pi}{4} = \frac{\pi}{12} AL \cdot DG \sqrt{(DE \cdot FG)} = \text{its solidity.}$$

The solidity of the whole cone ABC is

$$\frac{\pi}{12} BC^2 \cdot AK;$$

therefore, by the question,

(1) $$m + n : m :: BC^2 \cdot AK : AL \cdot DG \cdot \sqrt{(DE \cdot FG)}$$

The triangles DEG and AHL are similar; for, if a circle be described on AG as diameter, the points H and L will be in the circumference, and $AHL = AGL = DGE$, and $HLA = HGA = DEG$; hence

(2) $$DG \cdot AL = DE \cdot AH.$$

Combining equations (1) and (2)

(3) $\qquad m+n : m :: BC^2 . AK : DE . AH\sqrt{(DE . FG)}.$

Let $DE = 2x$, $FG = 2y$, $BC = 2b$, $AK = ab$; then $AI = ax$, $AH = ay$; and (3) becomes

$$m+n : m :: 4ab^3 : 4axy\sqrt{(xy)}. \qquad \therefore\ xy = b^2\left(\frac{m}{m+n}\right)^{\frac{1}{3}}.$$

Again, $DPI = DGQ = p$, and $GQ = a(y-x)$ and $DQ = x+y$.

$$\therefore\ a\tan p\,(y-x) = x+y; \quad \text{and} \quad x = b\left(\frac{m}{m+n}\right)^{\frac{1}{3}}\sqrt{\left(\frac{a\tan p-1}{a\tan p+1}\right)},$$

$$y = b\left(\frac{m}{m+n}\right)^{\frac{1}{3}}\sqrt{\left(\frac{a\tan p+1}{a\tan p-1}\right)}.$$

No satisfactory solution for the cases in which the section is a parabola or hyperbola has been received.

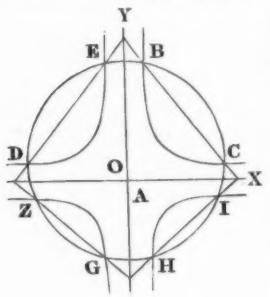
Problem IV. is pronounced to be wrong by several of the competitors.

Prize Solution of Problem V.

By Asher B. Evans, Madison University, Hamilton, N. Y.

Describe an ellipse having its foci at any points on one of the asymptotes of an equilateral hyperbola. The ellipse will cut each branch of the hyperbola and its conjugate in two real or imaginary points. Prove that a chord joining one pair of these points will be perpendicular to one of the other similar chords. Prove it first for a circle and then for an ellipse.

Take the asymptotes Ax and Ay as axes; and let x_1y_1, x_2y_2, x_3y_3, x_4y_4 represent B, C, Z, G, the four points common to the hyperbola and the circle; also let θ and θ_1 represent the angles which BC and GZ make with the axis of abscissas; then $\frac{y_2-y_1}{x_2-x_1} = \tan\theta$, and $\frac{y_4-y_3}{x_4-x_3} = \tan\theta_1$. The equation of the hyperbola is



(1) $\qquad xy = c^2;$

that of the circle is

$$x^2 + (y - a)^2 = r^2. \tag{2}$$

Combining (1) and (2),

$$y^4 - 2ay^3 + (a^2 - r^2)y^2 + c^4 = 0. \tag{3}$$

The roots of (3) are y_1, y_2, y_3, y_4; hence, by the theory of equations,

$$y_1 y_2 y_3 y_4 = c^4. \tag{4}$$

The co-ordinates of B, C, Z, G must satisfy (1); this condition gives $\tan\theta = -\frac{c^2}{y_1 y_2}$, and $\tan\theta_1 = -\frac{c^2}{y_3 y_4}$.

$$\therefore \tan\theta \tan\theta_1 = \frac{c^4}{y_1 y_2 y_3 y_4} = 1,$$

because (4) exists. Hence $\sin\theta \sin\theta_1 = \cos\theta \cos\theta_1$,

or $$\sin\theta \sin\theta_1 - \cos\theta \cos\theta_1 = \cos(\theta + \theta_1) = 0.$$

$$\therefore \theta + \theta_1 = 90°.$$

If E, D, I, H be four points common to the conjugate hyperbola and the circle, it is evident from considerations of symmetry that ED and IH, BC and ZG, make equal angles with the axis of x, whenever BC and DE are at right angles respectively with HI and ZG. By substituting the equation of the ellipse for that of the circle in the above proof, it may similarly be shown that this property does *not* hold for the ellipse.

No solution of the problem as amended (see Editorial Items for November number) has been received.

Some of the competitors are requested to take more pains with their penmanship and the arrangement of their solutions.

Simon Newcomb,
W. P. G. Bartlett,
Truman Henry Safford.

EXPOSITION OF THE PROCESS OF MATHEMATICAL DEVELOPMENT.

By JOHN PATERSON, A. M., Albany, N. Y.

[Concluded from Page 61.]

12. THE synchronous and successive series of forces for the time $\psi\, 1_t$ are therefore $1\,\varphi^{IV}$, $1\,\psi\,\varphi'''$, $1\,\psi^2\,\varphi''$, $1\,\psi^3\,\varphi'$, $1\,\psi^4\,\varphi^\circ$, and $1\,\varphi^{IV}$, $2\,\psi\,\varphi'''$, $3\,\psi^2\,\varphi''$, $4\,\psi^3\,\varphi'$, $1\,\psi^4\,\varphi^\circ$. Applying the successive forces to a succeeding interval $h\,1_t$ of time, and reasoning as above with $\psi\, 1_t$, we get the following: —

$\psi\, 1_t$	$h\, 1_t$.
$1\,\varphi^{IV} = 1\,\varphi^{IV}$:	$\frac{1.2}{1} h\varphi'''$, $\frac{1.2.3}{1\ 2} h^2\varphi''$, $\frac{1.2.3.4}{1.2.3} h^3\varphi'$, $\frac{1.2.3.4.1}{1.2.3.4} h^4\varphi^\circ = 1\,h^4\varphi^\circ$.
$1\,\psi\,\varphi''' = 2\,\psi\,\varphi'''$:	$\frac{2.3}{1}\psi h\varphi''$, $\frac{2.3.4}{1.2}\psi h^2\varphi'$, $\frac{2.3.4.1}{1.2.3}\psi h^3\varphi^\circ = 4\,\psi h^3\varphi^\circ$.
$1\,\psi^2\varphi'' = 3\,\psi^2\varphi''$:	$\frac{3.4}{1}\psi^2 h\varphi'$, $\frac{3.4.1}{1.2}\psi^2 h^2\varphi^\circ = 6\,\psi^2 h^2\varphi^\circ$.
$1\,\psi^3\varphi' = 4\,\psi^3\varphi'$:	$\frac{4.1}{1}\psi^3 h\varphi^\circ = 4\,\psi^3 h\varphi^\circ$.
$1\,\psi^4\varphi^\circ = 1\,\psi^4\varphi^\circ$.	

Thus $(\psi^4 + 4\,\psi^3\,h + 6\,\psi^2\,h^2 + 4\,\psi\,h^3 + h^4)\,\varphi^\circ = (\psi + h)^4\,\varphi^\circ$ is the final result generated by the force $1\,\varphi^{IV}$ in the time $(\psi + h)\ 1_t$.

13. So far the exposition has treated of free operations only, no restriction whatever being applied to the generating forces, and the result is the normal form of development. The transition to other forms, such as $(\psi \pm h)^{\pm n}$, $(\psi \pm h)^{\pm\frac{p}{q}}$, $a^{\pm\psi}$, $e^{\pm\alpha-\beta\psi}$, etc., etc., is effected by the introduction of a suitable condition in each case at the outset of the process.

14. Forces of the first order φ', originating from different sources, and entering into combination, produce effects or phenomena adapted to observation, the admeasurement of some simple forms of which may be the subject of a future article; and the present one will conclude with an application of the preceding principles to the demonstration of a result hitherto obtained by a different method.

Let $n = 2$: then the force $1\,\varphi''$ is an independent primitive generator of the first order and second degree, agreeing in character with a *constant force* as usually denominated, such as is the force of gravity at the surface of the earth. This force $1\,\varphi''$, entering into combination with a reacting force of the same order and degree, which, in the case of the terrestrial force mentioned, exists in the form of a heavy body, for which 1_μ may be written as denoting a unit of mass, will generate force of the second order and first degree φ', in a first interval of time $\psi'\,1_\iota$, in such amount that, with its ultimate value remaining constant, φ' will produce m times the effect in the equal subsequent interval $\psi''\,1_\iota$ of time, that was generated by it in the time $\psi'\,1_\iota$ of its own genesis: this force of the first degree φ', in this case, is the velocity (the unit mark of which may be 1_λ) accruing to the body acted upon by the force of gravity; and the effect of this velocity is the distance described by the falling body, the unit mark of which may be 1_ι, and which here takes the place of the force zero φ°.

The constant force $1\,\varphi''$ generates velocity uniformly, the ultimate value of which is $m\,1_\lambda$ at the expiration of the time $\psi'\,1_\iota$: it is therefore $\frac{m}{2}.1_\lambda$ at the expiration of the time $\frac{1'}{2}.\psi'\,1_\iota$, with which value it would carry the body 1_μ the distance $\frac{m}{2}\psi^2 1_\iota$ in the time $\psi''\,1_\iota$, and consequently does carry it the distance $\frac{m}{4}.\psi^2 1_\iota$ in the time $\frac{1''}{2}.\psi'\,1_\iota$, and the mth part of this distance in time $\frac{1'}{2}.\psi'\,1_\iota$; which again is the distance 1_μ will be carried by the increment of velocity generated by $1\,\varphi''$ in the second half-interval of time $\frac{1''}{2}.\psi'\,1_\iota$. This brings the expression $\left(\frac{1}{4}+\frac{m}{4}+\frac{1}{4}\right)\psi^2\,1_\lambda$; which, when $\psi = 1$, gives the equation $\left(\frac{1}{4}+\frac{m}{4}+\frac{1}{4}\right)1_\lambda = 1\,.\,1_\lambda$, because the effect is to be units in the first unit of time, and the effect is a measure of its cause for that time. The equation gives $m = 2$, and we construct the dynamical triangle: —

$$\begin{array}{l|l} \psi\, 1_t. & h\, 1_t. \\ \hline 1\,\varphi''; & \frac{1.2}{1} h\, 1_\lambda, \frac{1.2.1}{1.2} h^2\, 1_t. \\ 2\,\psi\,.\,1_\lambda; & \frac{2.1}{1} \psi\, h\, 1_t. \\ 1\,\psi^2\,.\,1_t. & \end{array}$$

The total is $(1\,\psi^2 + 2\,\psi\,h + 1\,h^2)\,1_t = (\psi + h)^2\,1_t$, the square of the time.

But when the unit of time $1_t = 1$ second, and the unit of distance $1_l = 1$ inch, observation shows that the distance described by any body under the influence of gravity in the first unit of time is approximately 193 inches: then the coefficient of gravity will not be units, but 193, when the unit of time is one second, and the unit of distance is one inch; and the result becomes $(1\,\psi^2 + 2\,\psi\,h + h^2)$ 193 inches in the time $(\psi + h)$ 1".

15. The preceding paragraphs have shown the genesis of a series of forces or powers $2\,\psi\,\varphi^{n-1}$, $3\,\psi^2\,\varphi^{n-2}$, $4\,\psi^3\,\varphi^{n-3}$, . . . $(n-1)\,\psi^{n-2}\varphi''$, $n\,\psi^{n-1}\,\varphi'$, $1\,\psi^n\,\varphi^\circ$, by the primitive power $1\,\varphi^n$, in the time $\psi\,1_t$. The number n is integral and indefinite, and consequently the series may be prolonged *ad libitum* to the right, or in the positive direction; thus placing $1\,\psi^n\varphi^\circ$, the zero of force or power, the final phenomenal result, at any required distance below the primitive independent $1\,\varphi^n$. The numerical coefficients, the exponents of the time ψ, and the indices of the powers φ, all traverse the series by unit steps. Setting aside the *efficient* symbol φ, transition from any term to the next lower one is made by adding unity both to the numerical factor and to the exponent of the literal factor of the *coefficient* of the term selected; or indeed any term whatever is reached at once by adding or subtracting the number appropriate to the interval of transition.

16. Proceeding from right to left in the series (A), and subtracting any convenient number (say 7) from the coefficient and exponent

of the several terms, there arises the series (B), in which terms may be continued in both directions.

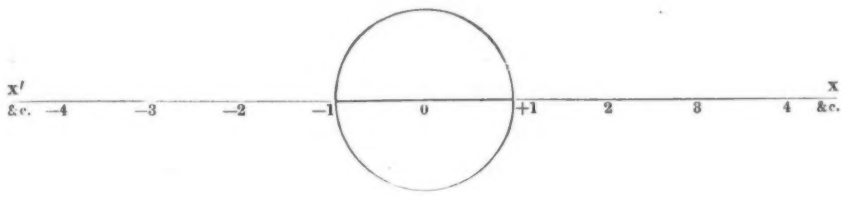
(A) $0\psi^{-1}, 1\psi^{0}, 2\psi^{1}, 3\psi^{2}, 4\psi^{3}, 5\psi^{4}, 6\psi^{5}, 7\psi^{6}, 8\psi^{7}, 9\psi^{8}, 10\psi^{9}, 11\psi^{10}, 12\psi^{11}, 13\psi^{12}$, &c.

(B) &c. $-6\psi^{-7}, -5\psi^{-6}, -4\psi^{-5}, -3\psi^{-4}, -2\psi^{-3}, -1\psi^{-2}, 0\psi^{-1}, 1\psi^{0}, 2\psi^{1}, 3\psi^{2}, 4\psi^{3}, 5\psi^{4}, 6\psi^{5}$, &c.

(C) &c. $1\psi^{-7}, 1\psi^{-6}, 1\psi^{-5}, 1\psi^{-4}, 1\psi^{-3}, 1\psi^{-2}, 1\psi^{-1}, 1\psi^{0}, 1\psi^{1}, 1\psi^{2}, 1\psi^{3}, 1\psi^{4}, 1\psi^{5}$, &c.

The series (B) is merely the series (A) prolonged also in the negative direction, and is therefore the general form of the series expressed in *successive* measures, to the terms of which correspond the *synchronous* measures expressed in the series (C). The coefficient 0 shows that $1\,\varphi^{n}$ is not generated, but must be assumed; and indeed $1\,\varphi^{n}$ is the unique primitive independent power, or *constant* generating force in mathesis, all beyond zero in (B) proceeding by indefinite regression towards negative infinity. The term $0\,\psi^{-1}$, which has $1\,\psi^{-1}\varphi^{\circ}$ for the measure of its synchronous value, marks and masks the transition from positive to negative coefficients; for when $\psi = 1$, the coefficient of the term $\frac{1}{1}\,\psi^{\circ}$ expresses the ratio of two equals, while $0\,.\,\frac{1}{1}\,\varphi^{n}$ indicates equality and opposition in the directions of the two forces which respectively generate the numerator and denominator. This transition from positivity to negativity, and *vice versa*, is effected by passing from linear through angular magnitude, to the opposite direction in space, by means of the revolving radius unity or 1_{r}, and comprises the region of logarithms and exponentials, circulars and elliptics, and many definite integrals, etc.

For the time $\psi\,1_{t} = 1\,.\,1_{t}$, the following diagram exhibits the measures of the coefficient in (B):—



17. That the relations of the terms of (B) are not changed by transition through the critical value, or point of discontinuity, 0, is shown by forming proportions between any two pairs of terms, as thus: —

$$5\,\psi^4 : 11\,\psi^{10} :: \quad 3\,\psi^2 \; : 9\,\psi^8, \; 14\,\psi^{12} = 14\,\psi^{12};$$

$$-2\,\psi^{-3} : \; 4\,\psi^3 \; :: -4\,\psi^{-5} : 2\,\psi, \quad 0\,\psi^{-2} = \; 0\,\psi^{-2};$$

$$-2\,\psi^3 : \; 4\,\psi^3 \; :: \quad 3\,\psi^2 \; : 9\,\psi^8, \; 7\,\psi^5 \; = \; 7\,\psi^5;$$

where the identical equations are deduced by adding together respectively the coefficients and the exponents of the means and of the extremes. From this it is inferred that under like operations, results deduced respectively from terms on the left and on the right of the zero in (B) are equally true.

18. The power zero φ° may be placed anywhere in the series (B), and then its coefficient must always be replaced by the corresponding term in (C). If φ° be placed to the right of the term $0\,\psi^{-1}$, the example always has an independent primitive $1\,\varphi^n$; for instance, the coefficient $3\,\psi^2$ corresponds to $1\,\psi^2\,\varphi^\circ$ as phenomenal result, and to $1\,\varphi''$ as independent primitive (which is a case furnished by nature in the constant force of gravitation). If $2\,\psi\,\varphi^\circ$ were selected, it corresponds to $1\,\psi\,\varphi^\circ$; that is, there is no force $2\,\psi\,\varphi$ generated, but the result of the genesis is $\psi\,\varphi^\circ = \psi\,1_l$, 1_l being the linear unit, or measure of the effect of a unit 1_λ of constant velocity in the unit of time 1_t, the independent primitive of which is $1\,\varphi' = 1_\lambda =$ a power of the first order. If $1\,\psi^0\,\varphi^\circ$ be selected, then $1\,\varphi^\circ = 1_l$ is not generated, but is an assumed unit of magnitude.

By these steps, it is seen that the most convenient method of counting will be to place φ° in (C), and, passing diagonally one term to the left in (B), proceed to the primitive $1\,\varphi^n$. Thus $1\,\psi^2\,\varphi^\circ$ recalls the generators $2\,\psi\,\varphi'$ and $1\,\varphi''$; $1\,\psi\,\varphi^\circ$ recalls $1\,\varphi'$; but while

$1\,\psi^0\,\varphi^\circ = 1\,\varphi^\circ$ may be assumed, it has also some kind of power $0\,\psi^{-1}\,\varphi$ for its generator; and as 0 is a measure of the ratio of equality, or a logarithm which expresses the number of times that one quantity is greater than another, its interpretation will appear in another place: $0\,\psi^{-1} = \frac{d\psi}{\psi} = d\,.\log\psi$.

The term $1\,\psi^{-1}\varphi^\circ$ calls up the series

$$-1\,\psi^{-2}\,\varphi',\ -2\,\psi^{-3}\,\varphi'',\ -3\,\psi^{-4}\,\varphi''',\ \ldots.$$

ad infinitum; and any term further to the left will evoke a similar series of negative coefficients and exponents, with efficients ascending indefinitely upwards from units in order, always commencing in (B) with φ' one step to the left of the term φ° selected in (C).

19. The scheme for the development of the function $(\psi + h)^n$ is as follows: —

$\psi\,1_\iota$:	$h\,1_\iota$.
$1\,\varphi^n$;	$\frac{1.2}{1}h\varphi^{n-1}$, $\frac{1.2.3}{1.2}h^2\varphi^{n-2}$, $\frac{1.2.3.4}{1.2.3}h^3\varphi^{n-3}$, $\frac{1.2.3.4.5}{1.2.3.4}h^4\,\varphi^{n-4}$, &c.
$2\,\psi\,\varphi^{n-1}$;	$\frac{2.3}{1}\psi\,h\,\varphi^{n-2}$, $\frac{2.3.4}{1.2}\psi\,h^2\,\varphi^{n-3}$, $\frac{2.3.4.5}{1.2.3}\psi\,h^3\,\varphi^{n-4}$, &c.
$3\psi^2\varphi^{n-2}$;	$\frac{3.4}{1}\psi^2\,h\,\varphi^{n-3}$, $\frac{3.4.5}{1.2}\psi^2\,h^2\,\varphi^{n-4}$, &c.
$4\psi^3\varphi^{n-3}$;	$\frac{4.5}{1}\psi^3\,h\,\varphi^{n-4}$, &c.
$5\psi^4\varphi^{n-4}$;	&c.
&c.	

On reaching the power $\varphi^{n-n} = \varphi^\circ$, all the terms of the lower diagonal series (the hypothenuse of the arithmetical triangle) are to be divided by the last factor of the numerators to get the final result.

Let $n = 4$: the triangle becomes

$\psi\, 1_t$.	$h\, 1_t$.			
$1\, \varphi^{IV}$;	$\frac{1.2}{1} h\, \varphi'''$,	$\frac{1.2.3}{1.2} h^2 \varphi''$,	$\frac{1.2.3.4}{1.2.3} h^3 \varphi'$,	$\frac{1.2.3.4.1}{1.2.3.4} h^4 \varphi^\circ = 1\, h^4 \varphi^\circ$.
$2\, \psi\, \varphi'''$;	$\frac{2.3}{1} \psi h\, \varphi''$,	$\frac{2.3.4}{1.2} \psi h^2 \varphi'$,	$\frac{2.3.4.1}{1.2.3} \psi h^3 \varphi^\circ = 4\, \psi h^3 \varphi^\circ$.	
$3\, \psi^2 \varphi''$;	$\frac{3.4}{1} \psi^2 h\, \varphi'$,	$\frac{3.4.1}{1.2} \psi^2 h^2 \varphi^\circ = 6\, \psi^2 h^2 \varphi^\circ$.		
$4\, \psi^3 \varphi'$;	$\frac{4.1}{1} \psi^3 h\, \varphi^\circ = 4\, \psi^3 h\, \varphi^\circ$.			
$1\, \psi^4 \varphi^\circ$.				

Which is $(\psi^4 + 4\, \psi^3 h + 6\, \psi^2 h^2 + 4\, \psi h^3 + h^4)\, \varphi^\circ = (\psi + h)^4\, \varphi^\circ$.

During the past time $\psi\, 1_t$, the independent primitive $1\, \varphi^{IV}$ of the fourth order has generated the final result $1\, \psi^4\, \varphi^\circ$, or has caused the unit of mass 1_μ to describe the distance $1\, \psi^4\, 1_l$ (or has generated the distance $1\, \psi^4\, 1_l$); and has also generated the forces or powers $4\, \psi^3\, \varphi'$, $3\, \psi^2\, \varphi''$, and $2\, \psi\, \varphi'''$, respectively of the first, second, and third orders, which become constant generators at the expiration of the time $\psi\, 1_t$. The power $4\, \psi^3\, \varphi'$ is a constant velocity $4\, \psi^3\, 1_\lambda$, which will of itself generate the distance $4\, \psi^3\, h\, 1_l$ in the future time $h\, 1_t$: it is the only velocity in existence in the system at the commencement of the time $h\, 1_t$, and is the differential coefficient of ψ^4, which gives the motion in the tangent of the angle that the curve makes with the axis of abscissa at the point ψ, rectangular co-ordinates understood. The power $3\, \psi^2\, \varphi''$ is a constant force which generates velocity uniformly; and as it has itself generated the velocity $4\, \psi^3\, 1_\lambda$ during its growth, it now adds the distance $6\, \psi^2\, h^2\, 1_l$ to the foregoing terms, by means of the velocity $12\, \psi^2\, h\, 1_\lambda$ which it generates in the time $h\, 1_t$. The power $2\, \psi\, \varphi'''$ is a constant force of the third order, which generates force of the second order uniformly, which generates velocity increasingly during the time $h\, 1_t$, and adds the distance $4\, \psi\, h^3\, 1_l$ to the foregoing terms. And lastly the primitive power $1\, \varphi^{IV}$ itself generates the distance $h^4\, 1_l$ in the time $h\, 1_t$, just as the distance $\psi^4\, 1_l$ was its final result in the time $\psi\, 1_t$.

20. Positive development begins at unity, proceeds towards positive infinity, and can be arrested at any term. Conversely, negative development begins at negative infinity, but may be taken up at any term, and concludes at unity. A closing portion of the scheme follows : —

$\psi 1_t$.	$h 1_t$.		
&c.	&c.	&c.	&c.
$-7\,\psi^{-8}\,\varphi^{\mathrm{VIII}}$;	$\frac{-7!}{-5!}\psi^{-8}h\,\varphi^{\mathrm{VI}}$,	$\frac{-7!}{-4!\,2!}\psi^{-8}h^2\,\varphi^{\mathrm{V}}$, ...	$\frac{-7!\,1}{7!}\psi^{-8}h^7\,\varphi^{\circ}$.
$-6\,\psi^{-7}\,\varphi^{\mathrm{VI}}$;	$\frac{-6!}{-4!}\psi^{-7}h\,\varphi^{\mathrm{V}}$,	$\frac{-6!}{-3!\,2!}\psi^{-7}h^2\,\varphi^{\mathrm{IV}}$, ...	$\frac{-6!\,1}{6!}\psi^{-7}h^6\,\varphi^{\circ}$.
$-5\,\psi^{-6}\,\varphi^{\mathrm{V}}$;	$\frac{-5!}{-3!}\psi^{-6}h\,\varphi^{\mathrm{IV}}$,	$\frac{-5!}{-2!\,2!}\psi^{-6}h^2\,\varphi'''$, ...	$\frac{-5!\,1}{5!}\psi^{-6}h^5\,\varphi^{\circ}$.
$-4\,\psi^{-5}\,\varphi^{\mathrm{IV}}$;	$\frac{-4!}{-2!}\psi^{-5}h\,\varphi'''$,	$\frac{-4!}{-1.2!}\psi^{-5}h^2\,\varphi''$,	$\frac{-4!}{3!}\psi^{-5}h^3\,\varphi'$, $\frac{-4!\,1}{4!}\psi^{-5}h^4\,\varphi^{\circ}$.
$-3\,\psi^{-4}\,\varphi'''$;	$\frac{(-3)(-2)}{1}\psi^{-4}h\,\varphi''$,	$\frac{-3!}{1.2}\psi^{-4}h^2\,\varphi'$,	$\frac{-3!\,1}{3!}\psi^{-4}h^3\,\varphi^{\circ}$.
$-2\,\psi^{-3}\,\varphi''$;	$\frac{(-2)(-1)}{1}\psi^{-3}h\,\varphi'$,	$\frac{(-2)(-1)}{1}\psi^{-3}h^2\,\varphi^{\circ}$.	
$-1\,\psi^{-2}\,\varphi'$;	$\frac{(-1)\,1}{1}\psi^{-2}h\,\varphi^{\circ}$.		
$+1\,\psi^{-1}\,\varphi^{\circ}$;			

in which the sign $n!$, or $\pm\frac{2n+1}{2}!$, as usual denotes the continued product $1.2....n$; or $\frac{1}{2}.\frac{3}{2}...\frac{2n+1}{2}$; or $(-\frac{1}{2})(-\frac{3}{2})...\left(-\frac{2n+1}{2}\right)$

The first diagonal line

$$\left(\frac{1}{\psi}-\frac{1}{\psi^2}h+\frac{1}{\psi^3}h^2-\frac{1}{\psi^4}h^3+\frac{1}{\psi^5}h^4-\frac{1}{\psi^6}h^5+\frac{1}{\psi^7}h^6-\frac{1}{\psi^8}h^7\right)\varphi^{\circ}$$

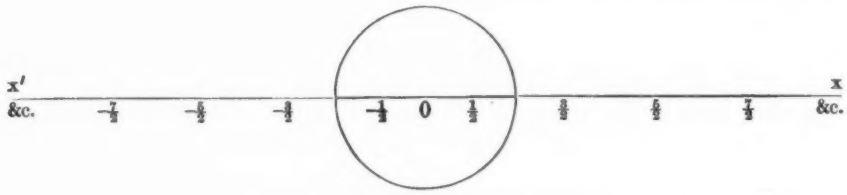
is the development of $(\psi+h)^{-1}$ carried through eight terms. But here, as in the case of positive development, the power zero φ° may be placed on any term of the first perpendicular line; where the

corresponding diagonal line, being divided throughout by the numerical factor common to every term, will express the development of $(\psi + h)$ to the index of ψ in the first term. For example, place φ° upon the fourth term from the last, and there appears: —

$\psi\, 1_t$.	$h\, 1_t$.			
&c.	&c.	&c.	&c.	
$-7\,\psi^{-8}\,\varphi^{IV}$;	$\frac{-7!}{-5!}\psi^{-8}\,h\,\varphi'''$,	$\frac{-7!}{-4!\;2!}\psi^{-8}\,h^2\,\varphi''$,	$\frac{-7!}{-3!\;3!}\psi^{-8}\,h^3\,\varphi'$,	$\frac{-7!\,1}{-3!\;4!}\psi^{-8}\,h^4\,\varphi^\circ$.
$-6\,\psi^{-7}\,\varphi'''$;	$\frac{-6!}{-4!}\psi^{-7}\,h\,\varphi''$,	$\frac{-6!}{-3!\;2!}\psi^{-7}\,h^2\,\varphi'$,	$\frac{-6!\,1}{-3!\;3!}\psi^{-7}\,h^3\,\varphi^\circ$.	
$-5\,\psi^{-6}\,\psi''$;	$\frac{(-5)\,(-4)}{1}\psi^{-6}\,h\,\varphi'$,	$\frac{(-5)(-4)\,1}{1\,.\,2}\psi^{-6}\,h^2\,\varphi^\circ$.		
$-4\,\psi^{-5}\,\varphi'$;	$\frac{(-4)\,1}{1}\psi^{-5}\,h\,\varphi^\circ$.			
$+1\,\psi^{-4}\,\varphi^\circ$.				

Total $\left(\frac{1}{\psi^4} - 4\frac{h}{\psi^5} + 10\frac{h^2}{\psi^6} - 20\frac{h^3}{\psi^7} + 35\frac{h^4}{\psi^8}\right)\varphi^\circ = (\psi + h)^{-4}\,\varphi^\circ$ to five terms.

21. To pass from integral to fractional development, observe, that retaining the unit interval 1_t between the several powers at the expiration of the first unit of time 1_t, they may be placed at fractional distances from the origin O or 0, as in Fig. 2. Subtract any



fractional number $\frac{p}{q}$, say $\frac{5}{2}$, from the coefficients and exponents of the terms of the successive integral series (B), and there arises the successive fractional series (D), to which corresponds the synchronous series (E): —

(B) &c. $-4\psi^{-5}$, $-3\psi^{-4}$, $-2\psi^{-3}$, $-1\psi^{-2}$, $0\psi^{-1}$, $1\psi^{0}$, $2\psi^{1}$, $3\psi^{2}$, $4\psi^{3}$, $5\psi^{4}$, $6\psi^{5}$, &c.

(D) &c. $-\frac{1}{2}^{3}\psi^{-\frac{15}{2}}$, $-\frac{1}{2}^{1}\psi^{-\frac{13}{2}}$, $-\frac{3}{2}\psi^{-\frac{11}{2}}$, $-\frac{7}{2}\psi^{-\frac{9}{2}}$, $-\frac{5}{2}\psi^{-\frac{7}{2}}$, $-\frac{3}{2}\psi^{-\frac{5}{2}}$, $-\frac{1}{2}\psi^{-\frac{3}{2}}$, $\frac{1}{2}\psi^{-\frac{1}{2}}$, $\frac{3}{2}\psi^{\frac{1}{2}}$, $\frac{5}{2}\psi^{\frac{3}{2}}$, $\frac{7}{2}\psi^{\frac{5}{2}}$, $\frac{9}{2}\psi^{\frac{7}{2}}$, &c.

(E) &c. $1\,\psi^{-\frac{15}{2}}$, $1\,\psi^{-\frac{13}{2}}$, $1\,\psi^{-\frac{11}{2}}$, $1\,\psi^{-\frac{9}{2}}$, $1\,\psi^{-\frac{7}{2}}$, $1\,\psi^{-\frac{5}{2}}$, $1\,\psi^{-\frac{3}{2}}$, $1\,\psi^{-\frac{1}{2}}$, $1\,\psi^{\frac{1}{2}}$, $1\,\psi^{\frac{3}{2}}$, $1\,\psi^{\frac{5}{2}}$, &c.

1°. Place the zero of power at the term $1\psi^{\frac{1}{2}}$, and get the triangle:

$\psi\,1_t$.	$h\,1_t$.			
&c.	&c.	&c.	&c.	
$-\frac{5}{2}\psi^{-\frac{1}{2}}\varphi^{IV}$;	$\frac{-\frac{5}{2}!}{-\frac{1}{2}.1}\psi^{-\frac{1}{2}}h\varphi'''$,	$\frac{-\frac{5}{2}!}{2!}\psi^{-\frac{1}{2}}h^2\varphi''$,	$\dots\frac{-\frac{5}{2}!\frac{1}{2}.1}{4!}\psi^{-\frac{1}{2}}h^4\varphi^{\circ}$	$=-\frac{1.1.3.5}{1.2.3.4}(\frac{1}{2})^4\frac{h^4}{\psi^{\frac{1}{2}}}\varphi^{\circ}$.
$-\frac{3}{2}\psi^{-\frac{1}{2}}\varphi'''$;	$\frac{-\frac{3}{2}!}{1}\psi^{-\frac{1}{2}}h\varphi''$,	$\frac{-\frac{3}{2}!\frac{1}{2}}{2!}\psi^{-\frac{1}{2}}h^2\varphi'$,	$\frac{-\frac{3}{2}!\frac{1}{2}.1}{3!}\psi^{-\frac{1}{2}}h^3\varphi^{\circ}$	$=+\frac{1.1.3}{3!}(\frac{1}{2})^3\frac{h^3}{\psi^{\frac{1}{2}}}\varphi^{\circ}$.
$-\frac{1}{2}\psi^{-\frac{1}{2}}\varphi''$;	$\frac{-\frac{1}{2}.\frac{1}{2}}{1}\psi^{-\frac{1}{2}}h\varphi'$,	$\frac{-\frac{1}{2}.\frac{1}{2}.1}{2!}\psi^{-\frac{1}{2}}h^2\varphi^{\circ}$		$=-\frac{1.1}{2!}(\frac{1}{2})^2\frac{h^2}{\psi^{-\frac{1}{2}}}\varphi^{\circ}$.
$+\frac{1}{2}\psi^{-\frac{1}{2}}\varphi'$;	$\frac{\frac{1}{2}.1}{1}\psi^{-\frac{1}{2}}h\varphi^{\circ}$			$=+\frac{1}{1}(\frac{1}{2})\frac{h}{\psi^{\frac{1}{2}}}\varphi^{\circ}$.
$1\,\psi^{\frac{1}{2}}\varphi^{\circ}$.				

Which is the development of $+\sqrt{(\psi+h)}$ to five terms.

2°. Place the power zero at the term $1\,\psi^{-\frac{1}{2}}$, and get

$\psi\,1_t$.	$1\,h_t$.			
&c.	&c.	&c.	&c.	
$-\frac{7}{2}\psi^{-\frac{9}{2}}\varphi^{IV}$;	$\frac{-\frac{7}{2}!}{-\frac{3}{2}!}\psi^{-\frac{9}{2}}h\varphi'''$,	$\dots\frac{-\frac{7}{2}!}{3!}\psi^{-\frac{9}{2}}h^3\varphi'$,	$\frac{-\frac{7}{2}!1}{4!}\psi^{-\frac{9}{2}}h^4\varphi^{\circ}$	$=+\frac{1.3.5.7}{4!}(\frac{1}{2})^4\frac{h^4}{\psi^{\frac{9}{2}}}\varphi^{\circ}$.
$-\frac{5}{2}\psi^{-\frac{7}{2}}\varphi'''$;	$\frac{-\frac{5}{2}!}{-\frac{1}{2}.1}\psi^{-\frac{7}{2}}h\varphi''$,	$\dots\frac{-\frac{5}{2}!}{2!}\psi^{-\frac{7}{2}}h^2\varphi'$,	$\frac{-\frac{5}{2}!1}{3!}\psi^{-\frac{7}{2}}h^3\varphi^{\circ}$	$=-\frac{1.3.5}{3!}(\frac{1}{2})^3\frac{h^3}{\psi^{\frac{7}{2}}}\varphi^{\circ}$.
$-\frac{3}{2}\psi^{-\frac{5}{2}}\varphi''$;	$\frac{-\frac{3}{2}!}{1}\psi^{-\frac{5}{2}}h\varphi'$,	$\frac{-\frac{3}{2}!1}{2!}\psi^{-\frac{5}{2}}h^2\varphi^{\circ}$		$=+\frac{1.3}{1.2}.(\frac{1}{2})^2\frac{h^2}{\psi^{\frac{5}{2}}}\varphi^{\circ}$.
$-\frac{1}{2}\psi^{-\frac{3}{2}}\varphi'$;	$\frac{-\frac{1}{2}.1}{1}\psi^{-\frac{3}{2}}h\varphi^{\circ}$			$=-\frac{1}{1}(\frac{1}{2})\frac{h}{\psi^{\frac{3}{2}}}\varphi^{\circ}$.
$1\,\psi^{-\frac{1}{2}}\varphi^{\circ}$.				

Which is the development of $-\sqrt{(\psi+h)}$ to five terms.

The development of $(\psi+h)^{\frac{1}{2}}$ is found as follows: —

$\psi 1_t$.	$h 1_t$.
&c.	&c. &c.
$-\frac{7}{2}\psi^{-\frac{9}{2}}\varphi^{\text{VIII}}$;	$\frac{-\frac{7}{2}!}{-\frac{3}{2}!}\psi^{-\frac{9}{2}}h\,\varphi^{\text{VII}}, \ldots \frac{-\frac{7}{2}!\frac{7}{2}!1}{8!}\psi^{-\frac{9}{2}}h^8\varphi^\circ = +\frac{7.5.3.1.1.3.5.7}{8!}(\frac{1}{2})^8\frac{h^8}{\psi^{\frac{9}{2}}}\varphi^\circ.$
$-\frac{5}{2}\psi^{-\frac{7}{2}}\varphi^{\text{VII}}$;	$\frac{-\frac{5}{2}!}{-\frac{1}{2}.1}\psi^{-\frac{7}{2}}h\,\varphi^{\text{VI}}, \ldots \frac{-\frac{5}{2}!\frac{7}{2}!1}{7!}\psi^{-\frac{7}{2}}h^7\varphi^\circ = -\frac{7.5.3.1.1.3.5}{7!}(\frac{1}{2})^7\frac{h^7}{\psi^{\frac{7}{2}}}\varphi^\circ.$
$-\frac{3}{2}\psi^{-\frac{5}{2}}\varphi^{\text{VI}}$;	$\frac{-\frac{3}{2}!}{1}\psi^{-\frac{5}{2}}h\,\varphi^{\text{V}}, \ldots \frac{-\frac{3}{2}!\frac{7}{2}!1}{6!}\psi^{-\frac{5}{2}}h^6\varphi^\circ = +\frac{7.5.3.1.1.3}{6!}(\frac{1}{2})^6\frac{h^6}{\psi^{\frac{5}{2}}}\varphi^\circ.$
$-\frac{1}{2}\psi^{-\frac{3}{2}}\varphi^{\text{V}}$;	$\frac{-\frac{1}{2}.\frac{1}{2}}{1}\psi^{-\frac{3}{2}}h\varphi^{\text{IV}}, \ldots \frac{-\frac{1}{2}.\frac{7}{2}!1}{5!}\psi^{-\frac{3}{2}}h^5\varphi^\circ = -\frac{7.5.3.1.1}{5!}(\frac{1}{2})^5\frac{h^5}{\psi^{\frac{3}{2}}}\varphi^\circ.$
$+\frac{1}{2}\psi^{-\frac{1}{2}}\varphi^{\text{IV}}$;	$\frac{\frac{3}{2}!}{1}\psi^{-\frac{1}{2}}h\,\varphi''', \ldots \frac{\frac{7}{2}!1}{4!}\psi^{-\frac{1}{2}}h^4\varphi^\circ = +\frac{7.5.3.1}{4!}(\frac{1}{2})^4\frac{h^4}{\psi^{\frac{1}{2}}}\varphi^\circ.$
$+\frac{3}{2}\psi^{\frac{1}{2}}\varphi'''$;	$\frac{\frac{5}{2}!}{\frac{1}{2}.1}\psi^{\frac{1}{2}}h\,\varphi'', \quad \frac{\frac{7}{2}!}{\frac{1}{2}.2!}\psi^{\frac{1}{2}}h^2\varphi', \quad \frac{\frac{7}{2}!1}{\frac{1}{2}.3!}\psi^{\frac{1}{2}}h^3\varphi^\circ = +\frac{7.5.3}{3!}(\frac{1}{2})^3\psi^{\frac{1}{2}}h^3\varphi^\circ.$
$+\frac{5}{2}\psi^{\frac{3}{2}}\varphi''$;	$\frac{\frac{7}{2}!}{\frac{3}{2}!}\psi^{\frac{3}{2}}h\,\varphi', \quad \frac{\frac{7}{2}!1}{\frac{3}{2}!2!}\psi^{\frac{3}{2}}h^2\varphi^\circ = +\frac{7.5}{1.2}(\frac{1}{2})^2\psi^{\frac{3}{2}}h^2\varphi^\circ.$
$+\frac{7}{2}\psi^{\frac{5}{2}}\varphi'$;	$\frac{\frac{7}{2}.1}{1}\psi^{\frac{5}{2}}h\,\varphi^\circ = +\frac{7}{1}(\frac{1}{2})\psi^{\frac{5}{2}}h\varphi^\circ.$
$1\,\varphi^{\frac{7}{2}}\varphi^\circ$.	

Thus positive fractional development is not arrested, but passes over zero by revolution.

22. When h is negative, the future time $h 1_t$, after the expiration of the time $\psi 1_t$, is changed for past time $-h 1_t$, the results of which are to be subtracted from those of the full time $\psi 1_t$. The factors of the numerators, even when under the negative sign, always tend in the positive direction; those of the denominators also advance for time *future* to $\psi 1_t$; consequently they will reverse their direction for time that is *past* when $\psi 1_t$ has expired. Then for the development of $(\psi - h)^4$, the form will be

$\psi 1_t$.	$h 1_t$.
$1\,\varphi^{\text{IV}}$;	$\frac{2!}{-1}h\,\varphi''', \quad \frac{3!}{-2!}h^2\varphi'', \quad \frac{4!}{-3!}h^3\varphi', \quad \frac{4!1}{-4!}h^4\varphi^\circ = +1\,h^4\varphi^\circ.$
$2\,\psi^2\varphi'''$;	$\frac{3!}{-1}\psi h\,\varphi'', \quad \frac{4!}{-2!}\psi h^2\varphi', \quad \frac{4!1}{-3!}\psi h^3\varphi^\circ = -4\,\psi h^3\varphi^\circ.$
$3\,\psi^2\varphi''$;	$\frac{4!}{-1.2!}\psi^2 h\,\varphi', \quad \frac{4!1}{-2!2!}\psi^2 h^2\varphi^\circ = +6\,\psi^2 h^2\varphi^\circ.$
$4\,\psi^3\varphi'$;	$\frac{4.1}{-1}\psi^3 h\,\varphi^\circ = -4\,\psi^3 h\,\varphi^\circ.$
$1\,\psi^4\varphi^\circ$.	

And for $(\psi - h)^{-4}$, the form will be

$\psi\, 1_t$.	$h\, 1_t$.		
&c.	&c.	&c.	&c.
$-7\,\psi^{-8}\varphi^{IV}$;	$\frac{-7\,!}{-5\,!\,-1}\psi^{-8}h\varphi'''$,	$\frac{-7\,!}{-4\,!\,-2\,!}\psi^{-8}h^2\varphi''$, ...	$\frac{-7\,!\,1}{-3\,!\,-4\,!}\psi^{-8}h^4\varphi^\circ = +\frac{35\,h^4}{\psi^8}\varphi^\circ$.
$-6\,\psi^{-7}\varphi'''$;	$\frac{-6\,!}{-3\,!\,-1}\psi^{-7}h\varphi''$,	$\frac{-6\,!}{-3\,!\,-2\,!}\psi^{-7}h^2\varphi'$,	$\frac{-6\,!\,1}{-2\,!\,-3\,!}\psi^{-7}h^3\varphi^\circ = +\frac{20\,h^3}{\psi^7}\varphi^\circ$.
$-5\,\psi^{-6}\varphi''$;	$\frac{-5\,!}{-3\,!\,-1}\psi^{-6}h\varphi'$,	$\frac{-5\,!\,1}{-3\,!\,-2\,!}\psi^{-6}h^2\varphi^\circ = +\frac{10\,h^2}{\psi^6}\varphi^\circ$.	
$-4\,\psi^{-5}\varphi'$;	$\frac{-4\,.\,1}{-1}\psi^{-5}h\varphi^\circ = +\frac{4\,h}{\psi^5}\varphi^\circ$.		
$+1\,\psi^{-4}\varphi^\circ$			

When the exponent of ψ is negative, uniform semicircular rotation commences with the beginning of the time $\psi\, 1_t$; when the coefficient of h is negative, an equal rotation begins with the time $h\, 1_t$.

NOTES ON PROBABILITIES.

By Simon Newcomb, Cambridge, Mass.

The problem with which the article on probabilities, in the Mathematical Monthly for May, 1860, was closed, belongs to a class which has been much discussed. We shall here repeat it, and then show a variation in it which leads to a result, which, at first sight, seems paradoxical.

An urn contains 100 balls, 1 of which is black, and 99 white; an individual, who falsifies one time in ten, draws a ball, and asserts that it is black. What is the probability that he tells the truth? We have found that it is $\frac{1}{12}$.

But suppose that the balls are numbered from 1 to 100, and that he asserts that the ball No. 79 is drawn. What is the probability that he tells the truth?

The solution is exhibited in tabular form, as follows. The first column contains the separate hypotheses which it is possible to make, column p gives the *a priori* probabilities of these hypotheses, and column q the probability of the observed event on the several hypotheses, as explained in the number of the Mathematical Monthly above referred to.

c	p	q	pq
No. 79 drawn	$\frac{1}{100}$	$\frac{9}{10}$	$\frac{9}{1000}$
Some other No. drawn,	$\frac{99}{100}$	$\frac{1}{990}$	$\frac{1}{1000}$

Whence the probability that No. 79 was drawn, or that the truth is told, is, by § 24, $\frac{\frac{9}{1000}}{\frac{9}{1000}+\frac{1}{1000}} = \frac{9}{10}$.

The value of q, on the second hypothesis, is obtained as follows: — Suppose that some other number than 79 is drawn. Then, if he is going to falsify, he has the 99 numbers which were not drawn, to choose from. Since there is supposed to be no reason why he should say 79 rather than any number, the probability that he will select this number from the 99 not drawn is $\frac{1}{99}$. But the probability that he will falsify in the first place is $\frac{1}{10}$. Whence the probability of saying 79 falsely is $\frac{1}{990}$.

Now the apparent paradox is, that the probability that he has told the truth is so vastly different when he says that a single black ball has been drawn from 100, from what it is when he says that the single number 79 has been so drawn, being $\frac{1}{12}$ in the first case, and $\frac{9}{10}$ in the second. Several forms of explanation may be given, and the reader can select that which seems to him most clear.

1. The difference of the results proceeds from the circumstance that he is supposed to have but two colors to choose from if he falsifies, viz. white and black, while he has 100 numbers. Since a white ball is drawn 99 times out of a hundred, and since he falsifies about one time in ten, he will have said black *about* 11 times in

every hundred, ten times falsely, and once truly. Whence I conclude that when he says black he does not tell the truth, for if I believed him to tell the truth every time, I should have to believe that an excessively improbable number of black balls had been drawn; instead of which I knew beforehand that he would make some ten or twelve false statements respecting the drawing of the black ball. But if 100 numbered balls have been drawn, his ten falsifications are supposed to have been divided amongst the 100 numbers I know not how, whence the chance that any one number was called falsely is only $\frac{1}{10}$.

If, however, I found that some one number, such as 79, had been called 10 or 12 times, I should conclude that the witness was disposed to falsify by saying 79, and thus I would not believe him, and the second case would be reduced to the first. Again, if there were a hundred colors from which he might choose, I would believe him when he said black, because I should conclude that if he were going to falsify, he would probably select some of the other 99 colors, and thus the first case of the problem would become identical with the second.

The following is an interesting problem, and might be useful. A bag contains an unknown number of tickets, numbered from one upwards. A person draws a ticket, notes the number, and replaces it; then repeats the operation till he has drawn s times. What judgment can he form as to the number of tickets in the bag?

Let p_n represent the *a priori* probability that the bag contains just n tickets, and no more. Let i be the largest number drawn in the s drawings. The number of tickets, then, cannot be less than i. We need not know any of the drawn numbers except the largest, because after we know this, every combination of smaller numbers will be equally probable on every admissible hypothesis, and will therefore be of no assistance in judging of these hypotheses. We

may then be supposed to know that s tickets have been drawn, and that the largest number observed is i. Now let us make our hypotheses. Suppose that the bag contains just i tickets. The probability that No. i would *not* be drawn on any one drawing is $\frac{i-1}{i}$. Hence the probability that it would not be drawn at all is $\frac{(i-1)^s}{i^s}$; hence the probability that it would be drawn on one or more trials is $\frac{i^s-(i-1)^s}{i^s}$. Again, if we suppose the bag to contain $i+h$ tickets, the probability that i would be the largest number drawn would be equal to the product of the probability that none of the h numbers above i would be drawn into the probability that i would be drawn if none of the h were drawn. The first of these probabilities is $\frac{i^s}{(i+h)^s}$; whence the probability that i would be the largest number is $\frac{i^s-(i-1)^s}{(i+h)^s}$. We can now arrange the quantities as before. We put for brevity $i^s-(i-1)^s=\triangle$.

c	p	q	pq
i balls	p_i	$\frac{\triangle}{i^s}$	$\frac{\triangle p_i}{i^s}$
$(i+1)$ balls	p_{i+1}	$\frac{\triangle}{(i+1)^s}$	$\frac{\triangle p_{i+1}}{(i+1)^s}$
$(i+2)$ balls	p_{i+2}	$\frac{\triangle}{(i+2)^s}$	$\frac{\triangle p_{i+2}}{(i+2)^s}$
&c.	&c.	&c.	&c.

Put P_n for the final probability that the bag contains just n tickets, and we have, by the general formula,

$$(1)\quad P_{i+h}=\frac{\triangle p_{i+h}}{(i+h)^s\left\{\frac{\triangle p_i}{i^s}+\frac{\triangle p_{i+}}{(i+1)^s}+\text{\&c.}\right\}}=\frac{p_{i+h}}{(i+h)^s\,.\,\Sigma},$$

putting, for brevity,

$$\Sigma=\frac{p_i}{i^s}+\frac{p_{i+1}}{(i+1)^s}+\frac{p_{i+2}}{(i+2)^s}+\cdots$$

By taking the sum of a sufficient number of the values of P_{i+h} from $h=0$ upwards to make $\frac{1}{2}$, we should find that number which

it was an even chance whether the balls would fall short of, or exceed. In getting the value of Σ, the series extends to all the numbers which it is possible that the bag should contain.

As a particular case, suppose that all numbers up to a certain limit are, before the drawing, equally probable. The values of p_n will then be all equal; and by putting

$$S = \frac{1}{i^s} + \frac{1}{(i+1)^s} + \frac{1}{(i+2)^s} + \text{\&c.},$$

for all possible values, we shall have

$$(2) \qquad P_i = \frac{1}{Si^s}, \quad P_{i+1} = \frac{1}{S(i+1)^s}, \ldots P_{i+h} = \frac{1}{S(i+h)^s}.$$

We see from these expressions that the highest number drawn is the most probable of *any single number*, so that if we were required to guess the exact number contained in the bag, we ought to guess this one. But if i and h are not very small, several numbers above i will each be almost as probable as i itself, and the guess would therefore probably be wrong. But it would be more satisfactory to find the probability that the number is contained between given limits. Thus, the probability that it does not exceed $i+k$ is, by summing the separate probabilities,

$$(3) \qquad \frac{1}{S}\left\{\frac{1}{i^s} + \frac{1}{(i+1)^s} + \frac{1}{(i+2)^s} + \ldots + \frac{1}{(i+k)^s}\right\}.$$

As a numerical example, suppose that all numbers are equally before drawing, and that upon drawing four times, the highest number drawn is 10. Then we have $i=10$, $s=4$, $h=\infty$.

Also $$S = \sum_1^\infty{}_x \frac{1}{x^4} - \sum_1^9{}_x \frac{1}{x^4} = \frac{\pi^4}{90} - \sum_1^9{}_x \frac{1}{x^4} = 0.000386650,$$

$$\frac{1}{i^s} = 0.0001 = \frac{1}{10^4}, \quad \frac{1}{(i+1)^s} = 0.0000683, \quad \frac{1}{(i+2)^s} = 0.0000482.$$

The sum of the first two numbers is rather more than $\frac{1}{2}S$, and

the sum of the first three rather greater. It is therefore rather less than an even chance that the bag contains 11 tickets, or less, but rather more than an even chance that it does not contain more than 12. If we make $k = 10$ in (3), the resulting value of the probability is $\frac{3}{3}\frac{4}{8}\frac{8}{6}\frac{1}{6}$. We might therefore wager more than 8 against 1 that the bag did not contain more than 20 balls.

As another numerical example, suppose that I read in a paper the addresses of ten persons, all at the same post-office, but having different boxes, of which the numbers are given. The highest number was 200. What is the probable number of boxes which the office contains, supposing that one box is as likely to be let to any person as another?

To solve this, we observe that the sum of the series

$$1 + \frac{1}{2^{10}} + \frac{1}{3^{10}} + \ldots + \frac{1}{(x-1)^{10}}$$

is

$$\frac{\pi^{10}}{93555} - \frac{1}{9x^9} - \frac{1}{2x^{10}} \cdot - \frac{10}{12x^{11}} + \frac{132}{72x^{13}} - \&c.$$

The sum from x to infinity is, therefore,

$$\frac{1}{x^{10}} + \frac{1}{(x+1)^{10}} + \frac{1}{(x+2)^{10}} + \ldots = \frac{1}{9x^9} + \frac{1}{2x^{10}} + \frac{10}{12x^{11}} - \frac{11}{6x^{13}} + \&c.$$

and by taking the difference of these values for different values of x, we can find the sum of the series between finite limits. Representing by $\Sigma_{i,i'}$ the sum of the series between the limits i and i' inclusive, we have

$$\Sigma_{200,\infty} = 22194, \qquad \Sigma_{200,215} = 11110,$$

in units of the twenty-sixth place of decimals. There is, therefore, about an even chance that the post-office does not contain more than 215 boxes.

When i is so large that we may neglect all terms after the first in the expressions for the sum of the series, we shall have

$$S = \Sigma_{i\infty} = \frac{1}{(S-1)\,i^{s-1}} \qquad \Sigma_{i,i'} = \frac{1}{(S-1)\,i^{s-1}} - \frac{1}{(S-1)\,(i'-1)^{s-1}};$$

which, from equation (3), gives for the probability that the number required is contained between the limits i and i' inclusive,

$$P_{i,i'} = 1 - \frac{i^{s-1}}{(i'+1)^{s-1}}.$$

If we wish to find the limits within which there is a given probability that the number is contained, we obtain from the last equation,

$$i' = \frac{i}{(1-P_{i,i})^{s-1}} - 1.$$

Mathematical Monthly Notices.

A Treatise on Astronomy, Descriptive, Theoretical, and Physical, Designed for Schools, Academies, and Private Students. By H. N. ROBINSON, A. M., Formerly Professor of Mathematics in the United States Navy; Author of a Treatise on Arithmetic, Algebra, Geometry, Trigonometry, Surveying, Calculus, Natural Philosophy, &c., &c. New York: Ivison & Phinney. 1858.

This treatise professes to give an exposition, not only of the operations of practical astronomy, but of the motions of the heavenly bodies as deduced from the law of gravitation.

A few extracts from the work would be amply sufficient to enable the professional astronomer to judge of the author's success in carrying out his plan, but as teachers and students generally will wish to form such a judgment, we shall add such comments as the extracts suggest.

On page 67 it is stated that, "The shape of the earth, thus ascertained by actual measurement, is just what theory would give to a body of water equal to our globe, and revolving on an axis in 24 hours." Unfortunately, the shape of our earth is very far indeed from being that which theory would give to a body of water "equal to our globe, and revolving on an axis in 24 hours," as any one will find by putting into numbers the results given by any writer on the figure of a homogeneous revolving fluid mass in equilibrium; e.g. DE PONTÉCOULANT, Système du Monde, Vol. II.

On page 79 and 80 the method given for computing the relation between the horizontal parallax and semidiameter of the moon is entirely erroneous in theory, from the *tangents* of those quantities being used instead of the *sines*. Practically, the mistake is of slight moment.

On page 89 we find the statement that the real velocity of the earth in the orbit "varies in the inverse ratio as the radius vector; *or it varies directly as the apparant diameter of the sun.*" Here the author has forgotten that the earth does not always move in a direction at right an-

gles to that of the line drawn through the earth and the sun; and his statement is not true, unless the velocity is first multiplied by the cosine of this angle.

The following explanation will strike the physicist as being both unique and original: —

"The inquiry is sometimes made why we do not have the warmest weather about the summer solstice, and the coldest weather about the time of the winter solstice.

"This would be the case if the sun immediately ceased to give extra warmth, on arriving at the summer solstice; but if it could radiate extra heat to warm the earth three weeks before it came to the solstice, it would give the same extra heat three weeks after; and the northern portion of the earth must continue to increase in temperature as long as the sun continues to radiate more than its medium degree of heat over the surface, at any particular place. Conversely, the whole region of country continues to grow cold as long as the sun radiates less than its mean annual degree of heat over that region. The medium degree of heat, for the whole year, and for all places, of course, takes place when the sun is on the equator; the average temperature, at the time of the two equinoxes. The medium degree of heat, for our northern summer, considering only two seasons in the year, takes place when the sun's declination is about 12 degrees north; and the medium degree of heat, for winter, takes place when the sun's declination is about 12 degrees south; and if this be true, the heat of summer will begin to decrease about the 20th of August, and the cold of winter must essentially abate on or about the 16th of February, in all northern latitudes." — p. 100.

We find some difficulty in making the different parts of this explanation seem consistent with each other; but it can hardly be necessary to tell any intelligent teacher of Natural Philosophy that no part of the explanation has any reference to the real cause of the phenomenon. May we ask how the author's theory would explain the fact that at the depth of twenty or thirty yards the temperature is highest in December and lowest in June?

In showing how to compute the equation of time, the author does not state whether the true or mean longitude of the sun is to be used in calculating that part of the equation of time which proceeds from "the variable inclination of the sun's motion to the equator." From the first explanation of the figure we should suppose he meant S, in fig. 22, p. 103, to represent the position of the mean sun; but he after speaks of ΥS as representing the sun's longitude, from which we might infer that he meant the true longitude to be used in finding the second correction.

On page 116 we find the statement that a planet appears stationary when moving nearly toward the earth, the author entirely forgetting that in the case supposed the planet would change its apparent position in consequence of the motion of the earth.

From page 142: "It [Mercury] is rarely visible, owing to its small size and proximity to the sun, and it never appears larger to the naked eye than a star of the fifth magnitude." Any one who has ever seen Mercury must know that it shines with nearly or quite the brilliancy of a star of the first magnitude; and it is frequently visible at its elongations, provided one takes the proper opportunity, and is not in too high a latitude.

On page 190 directions are given for finding the radius of the earth from the horizontal parallax of the moon, and the statement is made that by this means we can find the diameter of the earth to a greater degree of exactness than by any one actual measurement. The moon's horizontal parallax cannot be determined by observation to within a second of arc, that is, within about $\frac{1}{3600}$ of its whole value. Its cube is therefore subjected to an uncertainty of $\frac{1}{12000}$ of its whole value; and since the formula on p. 191 virtually contains the cube of the parallax as a factor, the final result is uncertain to the $\frac{1}{12000}$ part of its whole amount from this cause alone; and the diameter of the earth thus found may therefore be in error by six or seven miles. Yet it is said that "this result is more to be relied on than any other." We have said nothing about the uncertainty in the force of gravity, and the mass of the moon, both of which enter, or ought to enter, into the formula.

In § 186, pages 209 and 210, we read: —

"From the earliest ages the eccentricity of the orbit [of the earth] has been diminishing; and this diminution will probably continue until it is annihilated altogether, and the orbit becomes a circle; after which it will open out in another direction, again become eccentric, and increase in eccentricity to a certain moderate amount, and then again decrease.

"The period for these *vibrations*,' though calculable, *has never been calculated* further than to satisfy us that it is not to be reckoned by hundreds or even by thousands of years.' It is a period so long that the history of astronomy, and of the whole human race, is but a point in comparison."

To this we reply, that it is a well-known fact that neither the eccentricity of the orbit of the earth, or of that of any of the planets, will continue to diminish till the orbit becomes circular; and it is moreover a demonstrable fact that the orbits of none of the planets will ever become mathematically perfect circles. The statement in the second paragraph is made in the face of the fact that the periods, not exactly of "these *vibrations*," but of the actual vibrations, have been repeatedly computed and their reciprocals published by LAGRANGE, in the "Berlin Memoirs;" by JACOBI, in his "Mathematische Werke;" by PONTÉCOULANT, in the third volume of his "Théorie Analytique du Système du Monde;" and by LEVERRIER, in the "Annales de l'Observatoire," Tom. II., and in the French "Connaissance des Temps."

On the very first page of the chapter devoted to planetary perturbations our author states that, "The direct effect of any of these bodies [the planets] on the position of the sun is absolutely insensible."

Now the actual fact is, that the effects of the attraction of the planets on the sun is always nearly as great as on each other, and sometimes so much greater that all the principal terms proceed from this cause.

"Students in Astronomy very naturally infer that the attractions of Venus and Jupiter (when those planets are nearest to the earth) affect the longitude of the moon. Indeed, some have been so confident of this as to infer that some of the lunar equations were the results of such planetary attractions. But we assure them it is not so; none of the lunar equations refer to any other disturbing body than the sun. Yet all other bodies do disturb the lunar motion, but the amount of other disturbing forces is absolutely insensible, as we shall show by the following analysis:" —

The result arrived at is, that the moon's longitude is not affected by either Venus or Jupiter by more than the "very small fraction of a second;" and the discussion, about an octavo page in length, closes with, "Therefore the perturbations produced on the moon's motion by the planets are not, and need not be, taken into account." (*Lunar Perturbations: A Fragment*, pp. 351, 352.)

Now the facts are, that some ten years ago the Astronomer Royal of England found from observation, and PROF. HANSEN from theory, that Venus produces an inequality of 20″ or more in the longitude of the moon; and in HANSEN'S *Lunar Tables* the sum of the terms depending on Venus may amount to nearly 40″, and even a few terms depending upon Mars and Jupiter, though small, are retained. It will be seen, by any one who will take the trouble to examine PROF. PEIRCE'S *Tables of the Moon*, that the effect of Venus is not omitted in the Lunar Ephemeris, printed in the *American Ephemeris and Nautical Almanac*. The author supposes that the problem of perturbations is solved by simply computing the intensity of the force at any instant; and we do not find even an intimation that this force is a function of the time, and that an integration is necessary to determine what change it may produce in the lapse of centuries. Indeed, the discussion shows only the first faint conceptions of the problem. But we must pass over, for want of space, numerous other and equally flagrant and unpardonable errors into which the author has fallen. That the author of a text-book on Astronomy should be so unacquainted with the modern history and present state of the science is most extraordinary; and that he should misapprehend the character of its problems is hardly less so. We do not contend that the author of an elementary treatise should be a profound and original investigator in the science of which he treats; but we do insist that he should be able to write with ordinary accuracy; that he should have an accurate knowledge of the present state of the

science; and that he should have a clear comprehension of the general principles upon which the science is based. We regret to say that our author has lamentably failed in all these necessary particulars.

Introductory Course of Natural Philosophy, for the use of Schools and Academies. Edited from GANOT'S Popular Physics, by WILLIAM G. PECK, A. M., Professor of Mathematics, Columbia College, New York. Published by A. S. Barnes & Burr, 51 and 53 John Street, New York. 1860. pp. 480. 308 Illustrations.

During the past few years several excellent works on Physics have appeared in France. These works are marked by a good arrangement of the subjects treated, scientific accuracy, and clearness of statement combined with a large number of illustrations, which for beauty of design and execution have never been equalled. They are more particularly described by the term *experimental*, as distinguished from those in which the subjects are treated mathematically. On account of the acknowledged excellence of these works much use has been made of the matter and illustrations by authors preparing works on experimental Physics for the use of teachers and students in this country. Among the latest and best of them is one entitled *Cours de Physique, purement experiméntale, a l'usage des gens du monde, des aspirantes au brevet supérieur des élèves des écoles normales, des institutions de demoiselles, et en général, des personnes étrangères aux connaissances mathématiques, par* A. GANOT, Professeur de Physique; ouvrage orné de 308 magnifiques vignettes dessinées par GAGNIET, gravées sur bois par QUARTELEY, DUJARDIN, LAMBERT, et en relief, sur cuivre, par E. SALLE. We have given the title in full, to show that the author intended the work, not only for schools of certain grades, but *for the use of everybody;* and on this account, and so far as possible to avoid the necessity of apparatus, every principle is fully illustrated with copper-plate engravings far surpassing in design and execution anything ever before introduced into a text-book. We have never seen a *strictly popular work* on Physics, or Natural Philosophy, at all to be compared with it, and we sincerely thank the Editor and Publishers of the American edition for making the work more accessible to teachers and students in this country. The editor has made judicious use of the original work, and without producing an exact translation, has preserved its symmetry and better adapted it to the educational wants of this country; and the Publishers by illustrating their edition with *fac-simile* copies of the original engravings, by special arrangement with the author, M. GANOT, will have the satisfaction of knowing by its use that their efforts are appreciated. We are glad to see that the applications of fluids, steam, &c., as motive powers have not been introduced. Such an extension would have made the work too large and expensive for general use, and would not be in harmony with the plan of simply developing and explaining elementary principles. We take great pleasure in recommending this work to the attention of teachers and pupils, as well as the general reader, as being in all respects by far the best elementary one upon the subject which we have examined.

ERRATA. — Vol. III., p. 11, l. 7, for "where," read "whose"; p. 12, l. 3, for "or," read "and"; p. 14, l. 18, dele "even" after when; p. 16, last line, for $\frac{n(n-1)}{1.2} a^{n-1} x'$, read "$n a^{n-1} x'$;" p. 17, l. 10, for $\frac{1}{16a}$ read $\frac{1}{16a^2}$.

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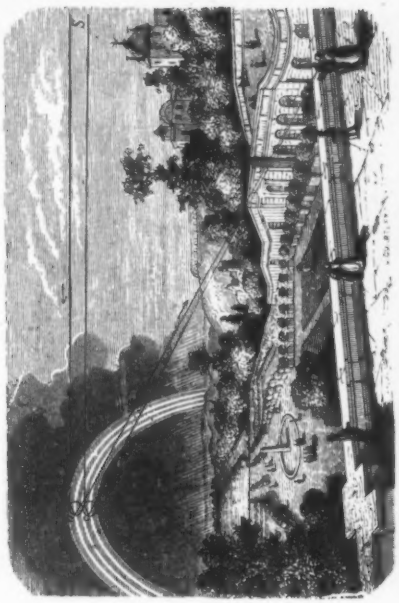

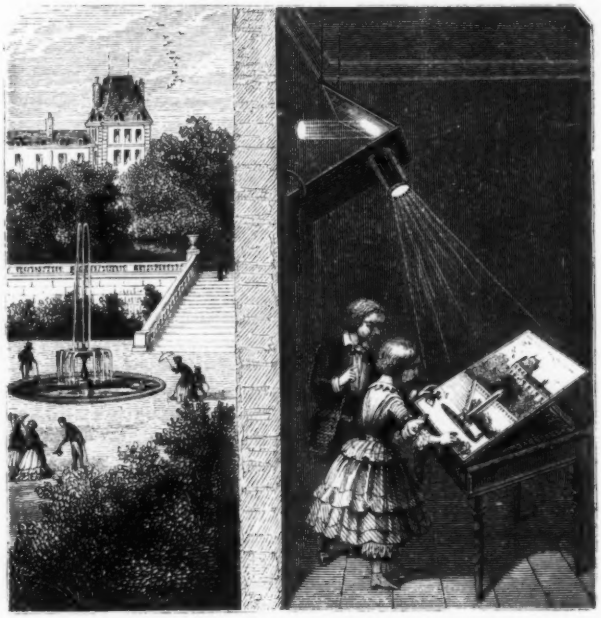

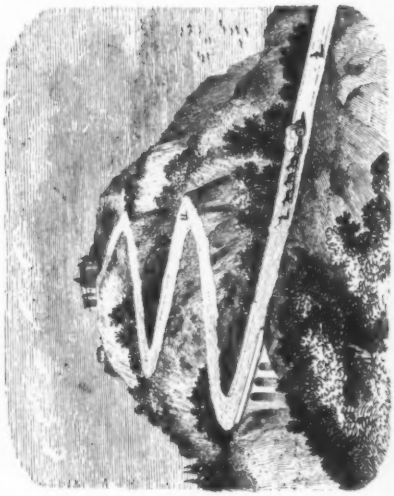

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THE NORTH AMERICAN REVIEW. No. CLXXXIX. — For October, 1860.

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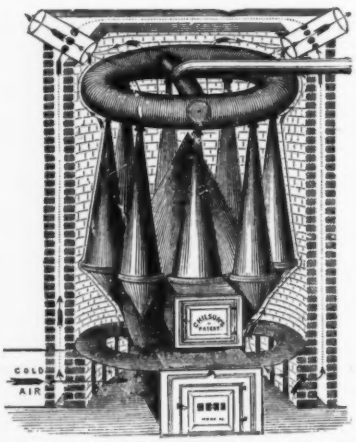

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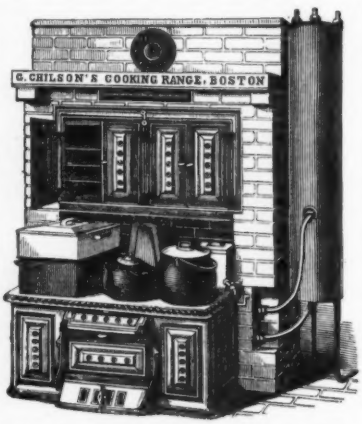

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